$$\mathrm{Ra}^* = \mathrm{Gr}\,\mathrm{Pr}\,\frac{\lambda_g}{\lambda_l}F_a, \tag{18}$$

$$F_a = \begin{cases} 15.38\,(0.64 - \beta\cdot 0.009552)/\Delta T, & 4 < \beta \leqslant 60, \\ 15.38\,(0.08 - \beta\cdot 0.000421)/\Delta T, & 60 < \beta < 200. \end{cases}$$

In conclusion we note that the observed appearance of natural convection in bubbles in foam accompanying weak heating could be of interest for accelerating interphase mass transfer in this system and deserves more detailed study.

NOTATION

λ_e, effective thermal conductivity of foam; λ_g, the thermal conductivity of air; λ_l, thermal conductivity of water; D, diameter of a foam cell; λ_{0e}, thermal conductivity of foam as $D \to 0$; $\varphi=\lambda_e/\lambda_{0e}$, correction factor; $Ra = GrPr\lambda_g/\lambda_l$, Rayleigh's number; $Gr = gD^3\beta_t\Delta T/\nu^2$, Grashoff's number; $Pr = \rho_g c_p \nu/\lambda_g$, Prandtl's number; ν, kinematic viscosity; β_t, thermal coefficient of expansion of air; Δt, temperature drop over one cell; g, acceleration of gravity; q, heat flux; β, foam ratio; ε, porosity; a, thermal diffusivity of the foam; T, temperature; ρ, density of the foam; φ_a and φ_λ, correction factors; A, coefficient; and c_p, heat capacity of air at constant pressure.

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METHOD FOR COMPREHENSIVE DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS AND AN ALGORITHM FOR COMPUTER ANALYSIS OF THE EXPERIMENTAL DATA

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UDC 536.2.021:022

The sequence of computer analysis of experimental data in comprehensive determination of the coefficients of thermal conductivity and thermal diffusivity in the Laplace transform domain is presented.

The difficulty of implementing most existing methods for experimental determination of the thermophysical characteristics (TPC) of materials [1] is linked with the complexity of the thermal processes occurring in the system consisting of the measuring cell and the sample of material tested as well as with the need for simple analytic expressions for calculating the coefficients from the experimental data. For this reason, a series of devices for establishing special conditions for heating the sample (maintaining constant or varying according to a definite law the temperatures and heat fluxes on the surfaces of the sample, one-dimensionality of the temperature field in the sample, etc.) are inserted into the experimental apparatus. An example of such methods are the methods of monotonic heating, developed and successfully implemented in [2]. The advantages of this apparatus include a wide temperature range, highly accurate determination of the TPC, and the possibility of evaluating the error in the results. However, the difficulty of setting up and calibrating the apparatus, owing to the fact that a large number of factors must be taken into account, requires that the

Kalinin Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 2, pp. 265-270, August, 1988. Original article submitted March 2, 1987.

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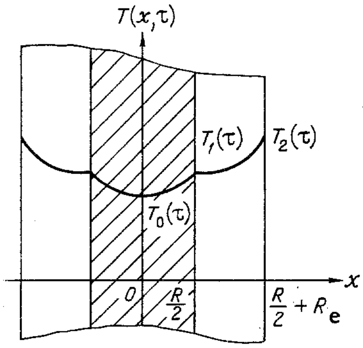

Fig. 1. Thermal scheme employed for measuring the thermal conductivity and thermal diffusivity.

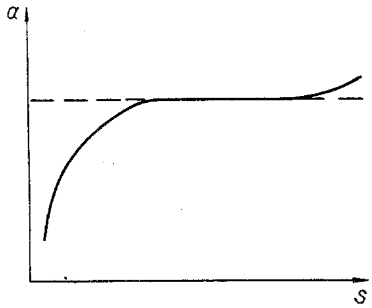

Fig. 2. Qualitative dependence of the coefficient a on the parameter s; the solid curve shows the reconstructed value of the coefficient a; the broken line is the true value of the coefficient a.

personnel using the equipment be highly qualified, and for this reason the equipment can be employed only in scientific and special studies.

The difficulties associated with the creation of special conditions for heating the sample can be avoided by employing the characteristics of more complicated heat transfer processes in determining the TPC. The methods based on the integral Laplace transformation (ILT) provide such a possibility. Thus a series of methods in which a monotonic, but arbitrarily varying temperature (or heat flux) is created on the surface of a sample was proposed in [3]. Under these conditions an ILT is employed and expressions are found in the transform domain for calculating the coefficients. The transforms of the experimental quantities (temperatures, heat fluxes), appearing in the working formulas, are calculated by integrating the experimental data. Other methods based on the ILT are also available [4, 5]. Aside from the advantages mentioned above, these methods also permit determining the confidence interval for the values of the coefficients sought.

Since the transforms cannot be calculated exactly because the measurements contain errors and the measurement time is finite, the practical implementation of the methods for determining the TPC in the transform domain requires analysis of the suitability of the mathematical model for describing the process, studies of the conditions required for a correct solution, finding the transform parameter that gives the minimum error in calculating transforms, and evaluation of the confidence interval for the TPC obtained.

We shall examine one method for comprehensive determination of the thermophysical characteristics. Let a system consisting of three infinite plates (Fig. 1) be heated symmetrically in a medium with a temperature higher than their starting temperature. The plate in the middle is the sample being tested, and the plates on either side consist of a material with known properties and are the reference plates.

The heat-transfer process is described by the system of equations with the boundary and initial conditions:

$$\frac{\partial T}{\partial \tau}=a\frac{\partial^2 T}{\partial x^2},\quad x\in\left[0,\frac{R}{2}\right],\ \tau>0,$$

$$\frac{\partial T_e}{\partial \tau}=a_e\frac{\partial^2 T_e}{\partial x^2},\quad x\in\left[\frac{R}{2},\ \frac{R}{2}+R_e\right],\ \tau>0,$$

$$T(x,\ 0)=T_e(x,\ 0),$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0,$$

$$\lambda\left.\frac{\partial T}{\partial x}\right|_{x=\frac{R}{2}}=\lambda_e\left.\frac{\partial T_e}{\partial x}\right|_{x=\frac{R}{2}},\tag{1}$$

$$T\left(\frac{R}{2},\ \tau\right)=T_e\left(\frac{R}{2},\ \tau\right),$$

$$T_e\left(\frac{R}{2}+R_e,\ \tau\right)=T_2(\tau).$$

Assume that the temperatures at points with the coordinates x = 0 and x = R/2 are known

$$T(0,\ \tau)=T_0(\tau),\ T\left(\frac{R}{2},\ \tau\right)=T_1(\tau),\tag{2}$$

and then, applying the ILT to the system (1) and taking into account the condition (2), we obtain the working formulas for determining the coefficients a and λ:

$$a=\frac{pR^2}{4\,\mathrm{Arch}^2\left[\frac{f_1(p)}{f_0(p)}\right]},\tag{3}$$

$$\lambda=\lambda_e\sqrt{\frac{a}{a_e}}\frac{f_1(p)}{\sqrt{f_1^2(p)-f_0^2(p)}\,\mathrm{sh}\sqrt{\frac{p}{a_e}}R_e}\left[\frac{f_2(p)}{f_1(p)}-\mathrm{ch}\sqrt{\frac{p}{a_e}}R_e\right].\tag{4}$$

As already pointed out the exact transforms of the time dependences of the temperatures cannot be found from the experimental data. For this reason the coefficients a and λ will be functions of the parameter $s = p\tau_{max}$ and a special analysis must be performed. First, the adequacy of the description of the real process by the mathematical model must be evaluated. A discrepancy can be caused by the following factors: the temperature dependence of coefficients, nonuniformity of the temperature field, asymmetry of the heating, the existence of contact thermal resistances, etc. As computer calculations showed, if the mathematical model corresponds to the real process, then the dependence of the coefficients a and λ on the parameter s is qualitatively of the same character as that shown in Fig. 2, i.e., the function has a horizontal section.

The condition for the existence of a solution follows from the structure of the formulas (3) and (4). The inequalities

$$f_1(p)>f_0(p),\ f_2(p)>f_1(p),\ \frac{f_2(p)}{f_1(p)}>\mathrm{ch}\sqrt{\frac{p}{a_e}}R_e$$

must hold. The conditions enumerated hold if the heating is monotonic, i.e., the rate of heating during the experiment must be positive.

The transform parameter p must be chosen based on the method for calculating transforms. There are two such methods: numerical integration and approximation of the experimental time dependences of the temperatures by some function. In the second method the function found is then transformed into the transform domain. In calculating the transforms by numerical integration quadratures of a higher algebraic degree of accuracy were employed in [3], this leads to the choice of orthogonal Chebyshev–Laguerre polynomials as the weighting functions. The times at which the measurements are performed are strictly fixed and are determined by the values of the corresponding roots of the Chebyshev–Laguerre polynomials. This leads, first, to the fact that the continuous dependence of the coefficients sought on the Laplace transform parameter cannot be analyzed and, second, to the fact that the "optimal" value of the

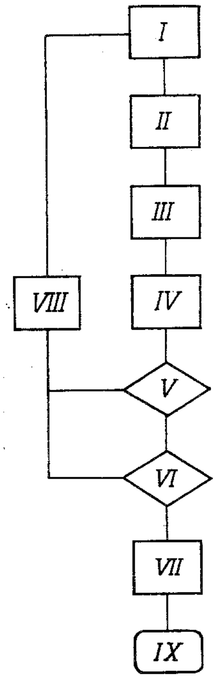

Fig. 3. Generalized algorithm for analyzing the experimental data: I) approximation of the experimental data with spline functions and evaluation of the average residual variance using the formula (7); II) calculation of a and λ based on the formulas (3) and (4) and approximation of the dependences $a = a(s)$, $\lambda = \lambda(s)$; III) finding the optimal value of the parameters s_a and s_λ from the conditions (6); IV) calculation of the optimal values of a and λ from the formulas (3) and (4); V) the condition $M > 1$; VI) the condition (10); VII) calculation of the confidence intervals for the coefficients a and λ sought; VIII) superposition of the random noise with the variance (7) on the temperature data; IX) stop.

parameter must be given a priori based on the error of the quadrature formula. The drawbacks described above can be eliminated by approximating the time dependences of the temperatures and then transforming them into the transform domain. It is best to choose cubic splines for the approximating functions. This made it possible to solve two problems: choosing the best, in the sense of minimum relative methodical error, values of the parameters s and s_λ and estimation of the confidence interval. Numerical analysis showed that the modulus of the derivative of the coefficient sought with respect to s is best used for selecting s_a and s_λ:

$$I_a = \left|\frac{\partial a}{\partial s}\right|, \quad I_\lambda = \left|\frac{\partial \lambda}{\partial s}\right|. \tag{5}$$

The optimal values of the parameters s_a and s_λ correspond to the conditions

$$I_a \to \min_s, \quad I_\lambda \to \min_s, \tag{6}$$

which is equivalent to vanishing of the second derivatives of the coefficients with respect to s. As experiments and numerical analysis showed, the methodical error of the approximation can be made negligibly small by varying the rate of heating, the duration of the experiment, and the time step of the measurements so as to achieve a horizontal section with an extent $\Delta s = 3$-4 (Fig. 2).

One of the chief problems in determining the TPC is to establish a relation between the error in the experimental measurement of the temperatures and the confidence interval containing the values of the coefficients sought. The average residual variance of the approximation of the experimental data with spline functions was adopted as the estimate of the random error in the temperature measurement:

$$\tilde{\sigma}_e^2 = \frac{1}{3(n-k-1)} \sum_{j=0}^{2} \sum_{i=1}^{n} (T_{ij} - \tilde{T}_{ij})^2, \tag{7}$$

where n is the number of measurements, k is the number of nodes in the spline, T_{ij} and $\tilde{T}_{ij}$ are, respectively, the experimental and approximating temperatures at the i-th point, j = 0, 1, 2.

The confidence interval was evaluated by statistical modeling using the method described in [5]. In so doing the random noise with the variance (7) is superposed on the spline functions approximating the experimentally determined temperatures. The mathematical expectation of the noise includes the errors owing to the uncertainty of the connection of the thermocouples, the asymmetry of the heating and of the system as a whole, and the existence of contact resistances. The coefficients are calculated next. Their variance at each step of the modeling is evaluated using a recurrence formula, which, in particular, has the following form for the thermal diffusivity:

$$\tilde{\sigma}^2_{a,M} = \tilde{\sigma}^2_{a,(M-1)} + \frac{1}{M-1}\left[(a_M - \bar{a}_M)^2 - \tilde{\sigma}^2_{a,(M-1)}\right], \tag{8}$$

where $\tilde{\sigma}^2_{a,M}$ is an estimate of the variance of the coefficient a at the step M; a_M is the value of the coefficient calculated at the step M; M = 1, 2, 3, ...,

$$\bar{a}_M = \bar{a}_{M-1} + \frac{1}{M}(a_M - \bar{a}_{M-1}). \tag{9}$$

It is obvious that $\bar{a}_1 = a_1$ and $\tilde{\sigma}^2_1 = 0$. Formulas analogous to (8) and (9) are employed to evaluate the variance of the coefficient λ.

The modeling is terminated when the condition

$$\max\left\{\frac{|\Delta\tilde{\sigma}^2_{a,M}|}{\tilde{\sigma}^2_{a,M}},\ \frac{|\Delta\tilde{\sigma}^2_{\lambda,M}|}{\tilde{\sigma}^2_{\lambda,M}}\right\} < \varepsilon, \tag{10}$$

where $|\Delta\tilde{\sigma}^2_{a,M}| = |\tilde{\sigma}^2_{a,M} - \tilde{\sigma}^2_{a,(M-1)}|$; $|\Delta\tilde{\sigma}^2_{\lambda,M}| = |\tilde{\sigma}^2_{\lambda,M} - \tilde{\sigma}^2_{\lambda,(M-1)}|$ and ε is a fixed small number (for example, $\varepsilon = 10^{-6}$), holds.

From the samples of a and λ obtained the following intervals are calculated [5]:

$$a = \bar{a} \pm 3\tilde{\sigma}^2_a, \tag{11}$$

$$\lambda = \bar{\lambda} \pm 3\tilde{\sigma}_\lambda \tag{12}$$

with a confidence probability of not less than 90%.

A generalized algorithm for analyzing the experimental data with an evaluation of the confidence interval is given in Fig. 3.

An automated experimental apparatus was developed for practical implementation of the proposed method. Polymethyl methacrylate, whose thermophysical properties are well known and remain virtually constant as a function of the temperature, was chosen as the reference in the measuring cell [2]. The temperature was measured with copper—constantan thermocouples with a diameter of $1 \cdot 10^{-4}$ m; the thermo-emf was measured with a Shch68003 digital voltmeter. The temperature drop across the reference and across the sample did not exceed 5 K in the experiments.

A package of programs was developed to analyze the experimental information presented in the form of a data file. The packet was realized based on UVK SM-4 in OS RAFOS-2.

In conclusion we shall present some data obtained on the apparatus developed: asbestos $a = (0.361 \pm 0.017) \cdot 10^{-6}$ m²/sec, $\lambda = (0.140 \pm 0.012)$ W/(m·K); ebonite $a = (0.105 \pm 0.004) \times 10^{-6}$ m²/sec, $\lambda = (0.152 \pm 0.013)$ W/(m·K); organic glass $a = (0.113 \pm 0.006) \cdot 10^{-6}$ m²/sec, $\lambda = (0.184 \pm 0.014)$ W/(m·K).

For insulation materials the relative error does not exceed 6% for the thermal diffusivity and 11% for the thermal conductivity.

NOTATION

λ and a, thermal conductivity and thermal diffusivity of the plate material; $T(x, \tau)$, temperature at a point with the coordinate x at time τ; R, thickness of the plate, p, Laplace transform parameter; $f_0(p)$, $f_1(p)$, $f_2(p)$, transforms of the temperature $T_0(\tau)$, $T_1(\tau)$, $T_2(\tau)$; τ_{max}, maximum time of the experiment. The index e refers to the standard.

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STRUCTURAL EFFECT OF THE TEMPERATURE COEFFICIENT OF RESISTIVITY OF ELECTRICALLY CONDUCTING HETEROGENEOUS SYSTEMS

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UDC 621.316.825:541.6:678.046

The temperature dependence of the electrical resistivity of heterogeneous filled systems on the thermomechanical properties of their components and the filler particle dimensions and the contact spots between them is derived and confirmed experimentally.

Heterogeneous electrically conducting systems that are a dielectric matrix filled with conducting particles are applied extensively in practice, consequently, their development and the investigation of their properties has received a great deal of attention [1-4]. For instance, lacquer—carbon black thick-film resistors, polymer current-conducting glues, and composites are utilized extensively in electrons and the radio industry. One of the most important characteristics of such systems is the temperature coefficient of the resistivity α determined from the formula

$$\alpha = \frac{1}{R}\frac{dR}{dt}. \tag{1}$$

An attempt at a mathematical description of the dependence of α on the mechanical-temperature constants of the conducting filler and binder is made in [5, 6] in an example of ceramic resistive composites:

$$\alpha = \frac{1}{T(1+mT)} - \frac{2}{T}, \tag{2}$$

where m is the coefficient characterizing the linear thermal expansion of a spherical filler particle in an elastic matrix and is a function of μ, β, E of the matrix and the filler.

It is assumed in the derivation of (2) that the centers of the particles in the conducting chains remain fixed as the temperature increases while α is determined by the change in resistivity of the hypothetical contact film between the particles. However, verification shows that the dependence (2) is inaccurate and does not reflect fully the processes proceeding in the contacts as the temperature changes. Indeed, if it is assumed that equality of the coefficient m to zero is achieved by selecting the heterogeneous system components by means of their mechanical-temperature constants, then evidently α should equal zero. According to (2), for $m = 0$ $\alpha = -1/T$.

In contrast to [5, 6], we examined a heterogeneous system model whose conducting particles make direct contact. According to [7], the particle resistivity in this case will be due mainly to contraction of the current lines of force at the contact spots whose area is

Institute of Physical Chemistry, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 2, pp. 270-275, August, 1988. Original article submitted April 15, 1987.